*Quantitative Reasoning Learning Progression Framework- Version 3-11-13*

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|  | **QR Progress Variable** | | |
| **Achievement Level** | **Quantification Act** | **Quantitative**  **Interpretation** | **Quantitative**  **Modeling** |
| **Level 4 (Upper Anchor)** | **4a** Variation: reasons about covariation of 2 or more variables; comparing, contrasting, relating variables in the context of problem  **4b** Quantitative Literacy: reasons with quantities to explain relationships between variables; proportional reasoning, numerical reasoning; extend to algebraic and higher math reasoning (MAA)  **4c** Context: situative view of QR within a community of practice (Shavelson); solves ill-defined problems in socio-political contexts using ad-hoc methods; informal reasoning within science context (Steen & Madison; Sadler & Zeidler)  **4d** Variable: mental construct for object within context including both attributes and measure (Thompson); capacity to communicate quantitative account of solution, decision, course of action within context | **4a** Trends: determine multiple types of trends including linear, power, and exponential trends; recognize and provide quantitative explanations of trends in model representation within context of problem  **4b** Predictions: makes predictions using covariation and provides a quantitative account which is applied within context of problem  **4c** Translation: translates between models; challenges quantitative variation between models as estimates or due to measurement error; identifies best model representing a context  **4d** Revision: revise models theoretically without data, evaluate competing models for possible combination (Schwarz) | **4a** Create Model: ability to create a model representing a context and apply it within context; use variety of quantitative methods to construct model including least squares, linearization, normal distribution, logarithmic, logistic growth, multivariate, simulation models  **4b** Refine Model: extend model to new situation; test and refine a model for internal consistency and coherence to evaluate scientific evidence, explanations, and results; (Duschl)  **4c** Model Reasoning: construct and use models spontaneously to assist own thinking, predict behavior in real-world, generate new questions about phenomena (Schwarz)  **4d** Statistical: conduct statistical inference to test hypothesis (Duschl) |
| **Level 3** | **3a** Variation: recognizes correlation between two variables without assuming causation, but provides a qualitative or isolated case account; lacks covariation  **3b** Quantitative Literacy: manipulates quantities to discover relationships; applies measure, numeracy, proportions, descriptive statistics  **3c** Context: display confidence with and cultural appreciation of mathematics within context; practical computation skills within context (Steen); lacks situative view  **3d** Variable: object within context is conceptualized so that the object has attributes, but weak measure (Thompson); capacity to communicate qualitative account of solution, decision, course of action within context, but weak quantitative account | **3a** Trends: recognize difference between linear vs. curvilinear growth; discuss both variables, providing a quantitative account  **3b** Predictions: makes predictions based on two variables, but relies on qualitative account; uses correlation but not covariation.  **3c** Translation: attempts to translate between models but struggles with comparison of quantitative elements; questions quantitative differences between models but provides erroneous qualitative accounts for differences  **3d** Revision: revise model to better fit evidence and improve explanatory power (Schwarz) | **3a** Create Model: create models for covariation situations that lack quantitative accounts; struggle to apply model within context or provide quantitative account  **3b** Refine Model: extend model based on supposition about data; do not fully verify fit to new situation  **3c** Model Reasoning: construct and use multiple models to explain phenomena, view models as tools supporting thinking, consider alternatives in constructing models (Schwarz)  **3d** Statistical: use descriptive statistics for central tendency and variation; make informal comparisons to address hypothesis |
| **Level 2** | **2a** Variation: sees dependence in relationship between two variables, provides only a qualitative account; lacks correlation, erroneously assumes causation  **2b** Quantitative Literacy: poor arithmetic ability interferes with manipulation of variables; struggle to compare or operate with variables  **2c** Context: lack confidence with or cultural appreciation of math within context; practical computation skills are not related to context  **2d** Variable: object within context is identified, but not fully conceptualized with attributes that are measurable; fails to communicate solution, decision, course of action within context; qualitative account without quantitative elements (Thompson) | **2a** Trends: identify and explain single case in model; recognize increasing/ decreasing trends but rely on qualitative account or change in only one variable  **2b** Predictions: makes predictions for models based on only one variable, provides only qualitative arguments supporting prediction  **2c** Translation: indicate preference for one model over another but do not translate between models ; acknowledge quantitative differences in models but do not compare  **2d** Revision: revise model based on authority rather than evidence, modify to improve clarity not explanatory power (Schwarz) | **2a** Create Model: constructs a table or data plot to organize two dimensional data ;create visual models to represent single variable data, such as statistical displays (pie charts, histograms)  **2b** Refine Model: extends a given model to account for dynamic change in model parameters; provides only a qualitative account  **2c** Model Reasoning: construct and use model to explain phenomena, means of communication rather than support for own thinking (Schwarz)  **2d** Statistical: calculates descriptive statistics for central tendency and variation but does not use to make informal comparisons to address hypothesis |
| **Level 1 (Lower Anchor)** | **1a** Variation: does not compare variables; works with only one variable when discussing trends,  **1b** Quantitative Literacy: fails to manipulate and calculate with variables to answer questions of change, discover patterns, and draw conclusions;  **1c** Context: does not relate quantities to context or exhibit computational skills  **1d** Variable, fail to relate model to context by identifying objects no attempt to conceptualize attributes that are measurable; discourse is force-dynamic; avoids quantitative account, provides weak qualitative account | **1a** Trends: do not identify trends in models  **1b** Predictions: avoids making predictions from models  **1c** Translation: fail to acknowledge two models can represent the same context  **1d** Revision: view models as fixed, test to see if good or bad replicas of phenomena (Schwarz) | **1a** Create Model: does not view science as model building and refining so does not attempt to construct models  **1b** Refine Model: accepts authority of model, does not see as needing refinement  **1c** Model Reasoning: construct and use models that are literal illustrations, model demonstrates for others not tool to generate new knowledge (Schwarz)  **1d** Statistical: does not use statistics; no calculation of even descriptive statistics |